PMM U.S.S.R.,Vol.45,pp.571-572

0021-8928/82/4 0571 $7.50/0

UDC 539.3

CYLINDRICAL SHELL SUBJECTED TO LATERAL DYNAMIC PRESSURE*

F.A. SOKOLOV

An elastic, circular, pressure-loaded, cylindrical shell under plane strain conditions is considered.

It is assumed that the pressure is a function of time and is expandable in a Fourier series of cosines of the angle. On the basis of the B.Z. Vlasov equations /1/, expressions are obtained for the displacements. An inaccuracy in /2/ is noted for the definition of the functions of time in terms of the series in the expressions for the displacements.

The equations of motion of a circular cylindrical shell under plane strain conditions /1/ will be written in the form

$$\frac{\partial^2 v}{\partial\theta^2}+\frac{\partial w}{\partial\theta}=\frac{\partial^2 v}{\partial\tau^2} \tag{1}$$

$$\frac{\partial v}{\partial\theta}+w+\alpha^2\left[\frac{\partial^2}{\partial\theta^2}\left(\frac{\partial^2 w}{\partial\theta^2}+w\right)+\frac{\partial^2 w}{\partial\theta^2}+w\right]=-\frac{\partial^2 w}{\partial\tau^2}+\frac{1}{H}q$$

$$\left(w=\frac{w'}{R},\quad v=\frac{v'}{R},\quad \tau=\left[\frac{E}{(1-\nu^2)\rho}\right]^{1/2}\frac{t}{R},\quad \alpha^2=\frac{h^2}{12R^2},\ H=\frac{E}{1-\nu^2}\frac{h}{R}\right)$$

Here w' and v' are the radial and tangential displacements, R and h are the shell radius and thickness, ρ is the density of the shell material, q is the pressure, and θ is the angular coordinate.

We examine the case when the shell is loaded by pressure in the form

$$q(\theta,\tau)=p(\tau)f(\theta),\quad f(\theta)=a_0+a_1\cos\theta+\sum_{n=2}^{\infty}a_n\cos n\theta$$

We consider the displacements and velocity to be zero for $\tau=0$. We apply a Laplace transform in time to the system (1) by denoting the transforms of the functions w, v, p by corresponding capitals, and s is the transformation parameter.

We seek the solution of the system of equations obtained after the Laplace transformation, in the form of Fourier series. We obtain

$$W=\frac{P(s)}{Hs}F_1(\theta,s),\quad V=\frac{P(s)}{Hs}F_2(\theta,s),\quad F_1(\theta,s)=\frac{sa_0}{s^2+1+\alpha^2}+\frac{s^2+1}{s(s^2+2)}a_1\cos\theta+\sum_{n=2}^{\infty}\frac{a_n(s^2+n^2)s}{\Delta}\cos n\theta$$

$$-F_2(\theta,s)=\frac{a_1}{s(s^2+2)}\sin\theta+\sum_{n=2}^{\infty}\frac{a_n sn}{\Delta}\sin n\theta,\quad \Delta=s^4+[n^2+1+\alpha^2(n^2-1)^2]s^2+n^2\alpha^2(n^2-1)^2$$

We obtain the following expression for the originals of the functions $F_i(\theta,s)$ (γ_n^2, β_n^2 are roots of the equation $\Delta=0$, taken with opposite sign)

$$f_1(\theta,\tau)=\frac{\sin(\tau\sqrt{1+\alpha^2})}{\sqrt{1+\alpha^2}}a_0+\left[\tau+\frac{\sin(\tau\sqrt{2})}{\sqrt{2}}\right]\frac{a_1}{2}\cos\theta+\sum_{n=2}^{\infty}\frac{a_n}{\beta_n^2-\gamma_n^2}\left[\left(\frac{n^2}{\gamma_n}-\gamma_n\right)\sin\gamma_n\tau-\left(\frac{n^2}{\beta_n}-\beta_n\right)\sin\beta_n\tau\right]\cos n\theta \tag{2}$$

$$-f_2(\theta,\tau)=\left[\tau-\frac{\sin(\tau\sqrt{2})}{\sqrt{2}}\right]\frac{a_1}{2}\sin\theta+\sum_{n=2}^{\infty}\frac{na_n}{\beta_n^2-\gamma_n^2}\left(\frac{\sin\gamma_n\tau}{\gamma_n}-\frac{\sin\beta_n\tau}{\beta_n}\right)\sin n\theta$$

---

*Prikl.Matem.Mekhan.,45,No.4,766-768,1981

Using the convolution theorem, we find expressions for the functions w, v.

$$w = \frac{1}{H}\int_0^\tau p(x) f_1(\theta, \tau - x)\,dx, \quad v = \frac{1}{H}\int_0^\tau p(x) f_2(\theta, \tau - x)\,dx$$

The solution of this problem is presented in /2/, where an error was allowed in determining the functions in the terms of series containing $\cos\theta, \sin\theta$.

We write the expressions obtained in /2/ for the displacements by taking account of the difference in the signs for the radial displacements and the pressure taken in the form /1,2/

$$w = \frac{1}{H}\int_0^\tau p(x) f_3(\theta, \tau - x)\,dx, \quad v = \frac{1}{H}\int_0^\tau p(x) f_4(\theta, \tau - x)\,dx \tag{3}$$

$$f_3(\theta, \tau) = a_0 \sin\tau - \frac{a_1}{\beta_1^2 - \gamma_1^2}\left[\left(\gamma_1 - \frac{1}{\gamma_1}\right)\sin\gamma_1\tau + \right.$$

$$\left.\left(\frac{1}{\beta_1} - \beta_1\right)\sin\beta_1\tau\right]\cos\theta + \sum_{n=2}^{\infty}\frac{a_n}{\beta_n^2 - \gamma_n^2}\left[\left(\frac{n^2}{\gamma_n} - \gamma_n\right)\sin\gamma_n\tau - \left(\frac{n^2}{\beta_n} - \beta_n\right)\sin\beta_n\tau\right]\cos n\theta$$

$$-f_4(\theta, \tau) = \frac{a_1}{\beta_1^2 - \gamma_1^2}\left(\frac{\sin\gamma_1\tau}{\gamma_1} - \frac{\sin\beta_1\tau}{\beta_1}\right)\sin\theta + \sum_{n=2}^{\infty}\frac{na_n}{\beta_n^2 - \gamma_n^2}\left(\frac{\sin\gamma_n\tau}{\gamma_n} - \frac{\sin\beta_n\tau}{\beta_n}\right)\sin n\theta$$

$$\gamma_n^2 = {}^1/_2\,(\alpha^2 n^4 + n^2 + 1) + {}^1/_2\,[\alpha^4 n^8 - 2\alpha^2 n^6 + (n^2 + 1)^2]^{1/2}$$

$$\beta_n^2 = {}^1/_2\,(\alpha^2 n^4 + n^2 + 1) - {}^1/_2\,[\alpha^4 n^8 - 2\alpha^2 n^6 + (n^2 + 1)^2]^{1/2}, \quad n \geqslant 1$$

The expressions (2) and (3) agree if the quantity α^2 is neglected in (2) in comparison to one, except for the terms containing $\cos\theta$ and $\sin\theta$. In (3) these terms should be the same as in (2).

This can be seen by solving the equation for the displacements /2/ for $n = 1$.

REFERENCES

1. VLASOV V.Z., General Theory of Shells. Selected Works, Vol.1, USSR Academy of Sciences Publishing House, Moscow, 1962.
2. HUMPHREYS J.S. and WINTER R., Dynamic response of a cylinder to a side pressure pulse, AIAA Journal, Vol.3, No.1, 1965.

Translated by M.D.F.